INVESTIGATING PRE-SERVICE TEACHERS' PROBLEM-SOLVING ABILITY AND THEIR CURRICULAR NOTICING ABILITY THROUGH PEDAGOGICAL SEQUENCING ACTIVITIES

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This study investigated how 155 pre-service teachers solved three pattern generalization problems in a two-part written test and sequenced them for teaching purposes to demonstrate their curricular noticing. Participants' solutions were analyzed using inductive content analysis, which showed that only 8.4% of PSTs produced correct answers to all three problems, and 14.2% proposed the same sequence as the researchers. As criteria for their sequencing, 52.9% used the problem's difficulty, variously associated with visuality, complexity, and feasibility, and 11% used their own understanding of how to solve the problem. It was also suggested that some participants' incorrect problem-solving influenced their curricular noticing. Implications for helping pre-service teachers improve their curricular noticing ability are discussed.

Keywords: pre-service teacher education, problem solving, teacher noticing

Introduction

The purpose of this study was to examine elementary pre-service teachers' (PSTs) problem solving abilities and their curricular noticing through problem-solving and problem-sequencing activities. With increased interest in curricular and pedagogical innovation in mathematics education, curricular noticing (Dietiker et al., 2018; Males et al., 2015), including proper sequencing of problems to scaffold students' understanding, has emerged as critical as noticing students' mathematical thinking (Dietiker et al., 2018; Males et al., 2015; Remillard, 2005). Thus, Males et al. (2015) extended the construct of noticing to teachers' treatment of curriculum materials and coined the term *curricular noticing*, defined as "the process through which teachers make sense of the complexity of content and pedagogical opportunities in written or digital curricular materials (p. 88)."

Teachers' ability to effectively use curriculum materials is a key component of teaching expertise and quality of instruction (Dietiker et al., 2018; Males et al., 2015; Remillard, 2005). For example, Amador et al. (2017) found that non-routine fractional tasks may support PSTs' interpretations of core mathematical properties of tasks they encounter in curricular materials. Also, Males et al. (2015) examined how PSTs engaged with curricular materials by asking them to create a lesson plan after providing Grade 6 teacher materials on the division of fractions from four curricular programs.

Although several studies have investigated PSTs' curricular noticing in fractions, research has rarely been conducted on pattern generalization, which has been emphasized as important for advancing learners' algebraic reasoning (Kaput, 1999; Radford, 2008). Also, as noted, Males et al. (2015) investigated PSTs' curricular noticing by having them create lesson plans with curricular materials from four sources provided by the researchers. However, PSTs may not be ready for noticing with such a range of curriculum materials. Rather, in the early stages of teacher preparation, they may need to develop a lens for curricular noticing in a controlled situation with specific tasks or activities.

In this study, the act of curricular noticing was channeled through sequencing three particular problems rather than through the provision of multiple curriculum materials. The following research questions guided this study: (1) How do the PSTs solve the three pattern generalization problems? (2) What reasoning do the PSTs think is required to solve each of the three problems? and (3) How do the PSTs sequence the three problems and rationalize their sequencing?

Theoretical perspectives: Curricular Noticing

Teacher noticing is one of the key teaching practices that PSTs should begin to develop in teacher education programs. Jacobs et al. (2010) described this construct as involving the three interrelated processes of attending to students' mathematical thinking, interpreting its meaning and importance, and making instructional decisions to further support their mathematical reasoning. In this construct, the first two components are required to attain the third component successfully, which is the core of noticing. Jacobs et al. termed this combination of interrelated processes as *professional noticing,* in that teacher's noticing has the specialized purpose of making appropriate pedagogical decisions. Dietiker et al. (2018) later expanded the professional noticing framework to include curricular noticing, consisting of three aspects, attending, interpreting, and responding. Attending involves "looking at, reading, and recognizing aspects of curricular materials," interpreting refers to "making sense of that to which the teacher attended," and responding requires "making curricular decisions based on the interpretation (e.g., generating a lesson plan, a visualization, or an enactment)" (p. 89).

As a framework in this study, we followed Dietiker et al.'s (2018) conceptualization of curricular noticing. However, because PSTs have few opportunities to engage in teaching in a classroom, we focused on one activity from each component of the curricular noticing framework to design a task using prompts corresponding to the three interrelated curricular noticing skills. We limited to asking PSTs to solve three problems to examine how they *perceived* the problems (Attending), to report what they thought was required to solve them to probe how they *comprehended* the problems (Interpreting), and to sequence the three problems with rationales to investigate how they would use them in *lesson planning* (Responding).

Methods

Data in this study comprised the responses to a written task. The 155 PSTs enrolled in several sections of a mathematics methods course at a large Southwestern university. The participants were enrolled in an elementary teacher education program leading to certification to teach from pre-K to eighth grade in the state where the data were collected. The PSTs had completed three mathematics content courses about number and operations, geometric reasoning, and algebraic reasoning before taking this methods course. The written task used in the study consisted of two parts (see Fig. 1). In the first part, the PSTs were asked to solve three pattern generalization problems developed by Stump et al. (2012) and to explain the reasoning/knowledge required to solve the problems. In the second part, they were asked to determine the proper sequence of the three problems to support student learning and to explain the rationale for their sequencing.

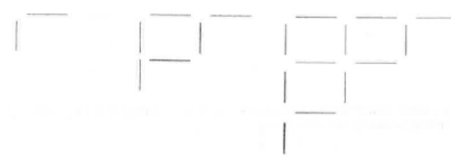
Part 1: Problem Solving
The following three problems are designed to help students learn to generalize pattern relationships. For each problem, (1) find an expression that you could use to find the *n*th term, and then (2) provide the kind of reasoning/knowledge required to solve each problem.

A. Hidden rectangle

Investigate the relationship between the number of rectangles that can be found in a row of squares and the length of the row by filling in the table. Remember that a square is a special kind of rectangle. For example, in the second figure, count each square and the rectangle formed from two squares, so there are three rectangles in the second figure.

B. Herringbone patterns

Shown below are the first three patterns in a sequence of herringbone designs. Investigate the relationship between the width of the pattern and the total number of line segments in the design by filling in the table.



C. Number of squares

Investigate the number of squares in each of the rectangles (including the hidden squares) by filling in the table. Find if there is a connection between the width of the rectangle and the total number of squares. For example, in the second figure, there are four 1x1 squares and one 2x2 square for a total of five squares.

Part 2: Sequencing the Three Problems

Revisit the three problems above and sequence the three problems in the best order for teaching upper elementary grade students (6-8th grade). Then provide your rationale for sequencing the three problems in that order.

Figure 1: Main Task of This Study

We applied an inductive content analysis approach (Grbich, 2007), which involved five processes: (a) reading each PST's response and creating codes based on the raw data, (b) identifying the correctness and themes of the responses, (c) creating categories and subcategories based on features of the PSTs' responses and solutions, (d) coding the categories and subcategories, and (e) interpreting the data quantitatively and qualitatively.

Findings

PSTs' Solutions

To probe how the PSTs solved the three pattern generalization problems, we examined whether they correctly identified the relationships and generalized expressions for the relationships they presented in their solutions. Table 1 shows the valid number sequence each problem contains, the generalized expressions for the number sequences identified, and the frequency of PSTs' correct/incorrect responses.

Table 1. Correctness of PSTs' Responses

Problem	number sequence	Generalized expression	Frequency (n = 155)		
			Correct	Incorrect	No attempt
Hidden rectangles	1, 3, 6, 10, 15…	$\frac{n^2+n}{2}$	13 (8.4 %)	129 (83.2 %)	13 (8.4 %)
Herringbone patterns	2, 6, 12, 20, 30…	n^2+n	88 (56.8 %)	52 (33.5 %)	15 (9.7 %)
Number of squares	2, 5, 8, 11, 24…	$3n-1$	51 (32.9 %)	97 (62.6 %)	7 (4.5 %)

As shown in the above table, there was a wide range of differences in correct and incorrect responses across the three problems. Only a small number of PSTs (8.4%) produced correct answers to all three problems. In particular, the incorrect responses were predominant in the hidden rectangles problem.

PSTs' Thoughts about Required Skills and Understanding

When asked about the necessary reasoning to solve each problem based on what they did to solve it, the PSTs proposed various ideas, from which three main themes emerged as common across all three problems: (a) knowing formulas, (b) explaining and identifying relationships, and (c) using problem-solving strategies. Also, generic statements and restating the generalized expression without additional explanations appeared across all three problems. For the hidden rectangles problem and the number of squares problem, geometry knowledge was additionally mentioned as a prerequisite for problem-solving. Table 2 shows PSTs' ideas about the required skills and understanding for solving the three problems.

Table 2. PSTs' Ideas about Required Skills and Understanding

	Examples	Hidden rectangles	Herring-bone patterns	Number of squares
Knowing formulas	• Use the formula • Remember the formula • Know the arithmetic sequence formula • Know the quadratic function formula	19 (12.3%)	21 (13.5%)	23 (14.8%)
Explaining /identifying relationships	• You must recognize that the number of rectangles is not increasing by a constant difference or ratio. • Each new rectangle adds 2 small squares and 1 hidden square, increasing by 3 each time.	41 (26.5%)	22 (14.2%)	11 (7.1%)
Problem-solving strategies	• Use a table • Trial and error: No distinct difference, so I made my own formula through trial and error. • Draw it out	11 (7.1%)	17 (11.0%)	13 (8.4%)
Generic statement without specifics	• Ability to find the pattern • Logical thinking • Algebra • Find relationships	27 (17.4%)	53 (34.2%)	24 (15.5%)

Simply Restating the generalized expression	• For (n + 1): Each place's answer is always that place x that place + 1 • For 3n + 1: Multiply 3 and add 1	20 (12.9%)	27 (17.4%)	45 (29.0%)
Geometry knowledge	• Need to know the definition of a square • Know that a square is also a rectangle • Need to see the rectangles inside rectangles	30 (19.4%)	0 (0 %)	24 (15.5%)
Other	• It is hard. • Unsure how to solve. • I don't know	0 (0 %)	4 (2.6%)	3 (1.9%)
No response		7 (4.5%)	11 (7.1%)	12 (7.7%)

PSTs' Suggested Problem Sequences and Rationales

PSTs were asked to sequence the three problems to support students' learning. Considering the cognitive demand and complexity, we agreed that the following is the proper sequence (C-B-A): C-Number of squares problem, B-Herringbone patterns problem, and A-Hidden rectangles problem. Table 3 shows the sequences PSTs proposed.

Table 3. Proposed Sequence by PSTs

	Problem A	Problem B	Problem C
First order	59 (38.1%) ***A***-B-C: 18 (11.6%) ***A***-C-B: 41 (26.5%)	9 (5.5%) ***B***-C-A: 4 (2.3%) ***B***-A-C: 5 (3.2%)	71 (45.8%) ***C***-A-B: 49 (31.6%) ***C***-B-A: 22 (14.2%)
Second order	54 (34.8%) B-***A***-C: 5 (3.2%) C-***A***-B: 49 (31.6%)	40 (25.8%) A-***B***-C: 18 (11.6%) C-***B***-A: 22 (14.2%)	45 (28.8%) A-***C***-B: 41 (26.5%) B-***C***-A: 4 (2.3%)
Third order	26 (16.5%) B-C-***A***: 4 (2.3%) C-B-***A***: 22 (14.2%)	90 (58.1%) A-C-***B***: 41 (26.5%) C-A-***B***: 49 (31.6%)	23 (14.8%) A-B-***C***: 18 (11.6%) B-A-***C***: 5 (3.2%)
No response	16 (10.3%)		

Twenty-two (14.2%) of the PSTs proposed the same sequence we proposed (C-B-A), including the 13 PSTs who solved all three problems correctly, whereas nine who produced the sequence of C-B-A did not solve all three problems correctly. Overall, Problem C was most frequently placed first in the recommended order, Problem A in second place, and Problem B in third.

The PSTs' justifications for their proposed sequencing of the problems are summarized in Table 4:

Table 4. PSTs' Justifications for Their Sequence

Category	Examples	Frequency
Difficulty	• Easier to harder (without explanation) • Easy: Start with the one that has a clear common difference • Easy: Start with the one that formula can be used • First, Problem C because it is easy to see the pattern, then	82 (52.9%)

	Problem B because it is easy to visualize, then Problem A because recursive reasoning can be confusing	
Scaffolding	• Problem A is the easiest, and Problem C is similar to Problem A • Because the first two build at each other and help them understand the third one.	20 (12.9%)
PSTs' own understanding/skills	• Starting with Problem C makes sense to me. I was able to understand this better than the others. • Problem B was complex for me, so I would not use it with students. • I will suggest the order in which I could complete them. • Problem B was more difficult for me, so I feel that the kids would also have difficulty understanding.	17 (11.0%)
Knowledge of geometry	• It requires knowing "squares are rectangles." • It is hard to find all of the hidden squares.	10 (6.5%)
Prior knowledge	• Use of known formula • Solving Problem B requires knowledge of exponents	6 (3.9%)
Importance	• Problem C is the most important	1 (0.6%)
General	• Critical thinking • Need to understand algebra	6 (3.9%)
No response		13 (8.4%)

More than 50% of the PSTs used the problem's difficulty as a criterion for the proposed sequence. However, we noted that the PSTs referred to the difficulty in many different ways, such as visuality, complexity, feasibility for directly applying formulas, and connections with multiple mathematical topics. We also noted that about 11% of the PSTs sequenced the problems based on their own understanding and skills, often mentioning what was easy or difficult for them. Although this category was also related to how the PSTs defined difficulty, we reported this rationale separately because these PSTs offered purely subjective interpretations based on their own abilities. Some PSTs also explained that Problems A and C were related because of their connection with geometry and juxtaposed them in the sequence. It is worth noting that the PSTs' incorrect problem-solving might have influenced their proposed sequence. For example, many PSTs produced incorrect answers by counting only 1x1 squares, resulting in straightforward arithmetic sequences, which they felt were easy.

Discussion and Implications

Gaps to Fill in PSTs' Understanding and Execution of the Problem

We noted that only a small number of PSTs (8.4%) found correct answers in all three problems. It is worth noting that these PSTs did not rely on the mechanical application of existing arithmetic or geometric formulas. Instead, they first looked for the problem's structure and generated patterns.

Some of our observations are worth further discussion when reviewing the incorrect responses. First, we found that most errors in the two problems (hidden rectangles problem and number of squares problem) occurred due to misunderstanding the problems. Specifically, when PSTs did not count all rectangles or squares (e.g., only counting 1x1 unit squares), they ended up with straightforward arithmetic sequences, such as the sequence with a common different 1 or 2. We intentionally used tasks that could create more than basic arithmetic or geometric sequences

where the common difference or common ratio was not apparent using geometric contexts. However, unlike our expectations, our tasks could not fully facilitate many PSTs' in-depth investigative work. We conjecture that many PSTs were familiar with working on typical number patterns neatly presented for directly applying formulas. Their prior experience of what and how they learned pattern generalization does matter. If PSTs' prior experiences in pattern generalization were mainly focused on a rule-driven approach, it would be necessary to provide opportunities for PSTs to investigate atypical number patterns that may integrate other domains of mathematics, such as geometry.

Second, many PSTs who could identify the correct number patterns still failed to present generalizable expressions. This result was particularly prevalent in the herringbone pattern problem (e.g., 75% of the incorrect cases). A few apparent strategies included guess and check, as the generalized expression only worked for some, not for all terms. Also, regardless of the correctly identified number patterns, some PSTs presented the general form of an arithmetic sequence, considering the difference between the first and the second terms as the common difference for the entire patterns, showing the application of formulas in a rote manner. In Common Core State Standards for Mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), students are expected to explore patterns using various modes of representations, such as physical materials, drawings, tables, words, and symbols, to make sense of the regularities and to see the changes quantitively and abstractly. If this kind of exploration was not a substantial part of PSTs' prior experiences, they need to have such experiences in the teacher education program.

Instructional Sequence: What Matters

When asked to sequence the three pattern generalization problems, we noted that a small number of PSTs (22 PSTs, 14.2%) suggested the instructional sequence of the problems that was the same as the researchers' sequence. Apparently, the PSTs who misunderstood the problems and yielded simple arithmetic sequences for the hidden rectangles problem and the number of squares problem were unable to sequence the problems properly. Of these 22 PSTs, 13 PSTs were those who solved all three problems correctly. This result implies that teachers' content knowledge is necessary, although we cannot say it is the only sufficient factor for the appropriate instructional decisions.

In examining PSTs' justifications for their proposed sequence, "difficulty" was the most predominant reason. However, we noted that there was no consensus on what it meant by "difficulty." While some PSTs simply said "from easier to harder" without explanations, other factors, such as visuality, complexity, the feasibility of direct application of arithmetic sequence formula, and the connection with other topics, were also addressed. It is also noted that about 11% of PSTs relied on their own problem-solving ability and comfort/confidence level in sequencing the problems, assuming that students' problem-solving ability and comfort/confidence level would be similar to theirs. Also, this tendency might hinder PSTs from eliciting and interpreting students' thinking and offering appropriate interventions as needed, especially when students' approaches are different from PSTs'. Thus, teacher educators need to provide more opportunities for PSTs to examine and analyze strategies different from theirs and decipher the reasoning behind those approaches. It is not our intention to suggest that there is a pre-defined instructional sequence, and that PSTs need to master the pre-determined sequence. Instead, we suggest PSTs need to be aware of various factors that should be considered in the

decision of instructional sequence and to build the ability to discern what factors should be foregrounded and backgrounded.

This study contributes to the current literature on curricular noticing and the knowledge base of teacher education. In particular, this study has implications for teacher educators working on designing mathematics education courses for PSTs and researchers interested in a better understanding teachers' knowledge and pedagogical strategies. That is, the findings of this study suggest that elementary mathematics teacher education programs need to include more curricular noticing activities in the form of sequencing tasks and other activities that enable PSTs to experience curricular noticing in the early stages of their teaching careers. This study also suggests the importance of further investigations of PSTs' curricular noticing ability as demonstrated in other mathematics teaching contexts.

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